Enthalpy based model of continuous casting process

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Abstract

Mathematical description and numerical model of continuous casting process is discussed. In particular the rectangular, vertical steel casting is considered. The energy equation and boundary-initial conditions are formulated using the enthalpy convention, at the same time the solidification model called the one domain method is taken into account. At the stage of numerical computations the finite difference method is applied and the choice of mesh step in the vertical direction results from the pulling rate, this approach assures the essential simplification of FDM algorithm. In the final part of the paper the examples of computations are shown.

Keywords: heat transfer, numerical analysis, finite difference methods

1. Governing equations

Equation describing the thermal processes in domain of continuous casting can be written in the form (only heat conduction is taken into account)

\[
C(T) \left[ \frac{\partial T(x,t)}{\partial t} + w \cdot \nabla T(x,t) \right] = \nabla \left[ \lambda(T) \nabla T(x,t) \right]
\]  

(1)

where \(C(T)\) is a substitute volumetric thermal capacity [1, 2, 3] (this parameter appears, as a rule, when the one domain method is applied), \(\lambda(T)\) is a thermal conductivity, \(w = [0, 0, w]\) is a pulling rate (a vertical plant is considered), \(T, x, t\) denote the temperature, spatial co-ordinates and time.

In a case of vertical, rectangular cast slab the equation (1) takes a form

\[
C(T) \left[ \frac{\partial T(x,t)}{\partial t} + w \frac{\partial T(x,t)}{\partial x_3} \right] = \sum_{e} \frac{\partial}{\partial x_e} \left[ \lambda(T) \frac{\partial T(x,t)}{\partial x_e} \right]
\]  

(2)

where \(x = \{x_1, x_2, x_3\}\) \((x_3\) is a pulling direction c.f. Figure 1).

\[\text{Figure 1: Domain considered and differential mesh}\]

Equation (2) is supplemented by the following boundary and initial conditions:

- on the lateral surface of casting

\[
- \lambda(T) \frac{\partial T(x,t)}{\partial x_e} = a(x_e)[T(x,t)-T_e], \quad e = 1 \cup 2
\]  

(3)

where \(a\) is a heat transfer coefficient (this parameter changes for successive sectors of cooling zones), \(T_e\) is a cooling water temperature,

- on the upper surface of casting

\[
T(x,t) = T_p
\]  

(4)

where \(T_p\) is a pouring temperature,

- on the conventionally assumed bottom surface of the system the no-flux condition should be taken into account,

- for time \(t = 0\) \(T(x,t) = T_r\), from the technological point of view a such condition is quite artificial, but the aim of computations is to find the results corresponding to the pseudo-steady thermal conditions and this solution will be correct.

Let us introduce the physical enthalpy related to an unit of volume, this means

\[
H(T) = \int_{T_r}^{T} C(\mu) d\mu
\]  

(5)

where \(T_r\) is a reference level.

It is easy to check that the energy equation written using the enthalpy function takes a form

\[
\frac{\partial H(x,t)}{\partial t} + w \cdot \nabla H(x,t) = \nabla \left[ a(H) \nabla H(x,t) \right]
\]  

(6)

and \(a = \lambda/C\). The boundary-initial conditions above presented can be in a natural way transformed to the enthalpy convention, except that the Robin boundary condition is a little different:

\[
-a(T) \frac{\partial H(x,t)}{\partial x_e} = a_H(x_e)[H(x,t)-H_e], \quad e = 1 \cup 2
\]  

(7)

where \(a_H = a/H_e\) and \(H_e\) is an integral mean of substitute thermal capacity in the interval \(T \in [T_r, T_e]\) \((T_r\) is a local
boundary temperature) [1]. At the stage of numerical computations this change of Robin conditions does cause the essential complications.

In Figure 2 the example of enthalpy function corresponding to the assumption that the substitute thermal capacity of material considered can be approximated by a piece-vise constant function (e.g. [1, 2, 3] is presented).

![Figure 2: Enthalpy function](image)

2. Numerical model

At the stage of computations the finite difference method in the version close to approach presented by Mochnacki and Suchy [1] is used. Let us assume that the mesh step in $x_3$ direction equals $h_3 = w \Delta t$ ($\Delta t$ is a time step).

Then the left hand side of energy equation can be approximated as follows

$$
\frac{\partial H}{\partial t} + \frac{\partial F}{\partial x_1} \approx \frac{H_{i,j}^{n+1} - H_{i,j}^{n}}{\Delta t} + \frac{H_{i,j}^{n+1} - H_{i,j-1}^{n+1}}{w \Delta t} = \frac{H_{i,j}^{n+1} - H_{i,j-1}^{n+1}}{\Delta t} \tag{8}
$$

The right hand side of equation discussed takes a form (internal nodes)

$$
[\nabla (a \nabla H)]_{i,j} \approx \frac{H_{i,j+1}^{n} - H_{i,j}^{n}}{R_{i,j+1}^{n}} \Phi_2 + \frac{H_{i+1,j}^{n} - H_{i,j}^{n}}{R_{i+1,j}^{n}} \Phi_3 + \frac{R_{i,j+1}^{n}}{R_{i+1,j}^{n}} \Phi_1 + \frac{R_{i,j}^{n}}{R_{i+1,j}^{n}} \Phi_1 \tag{9}
$$

where $s = f$ or $s = f + 1$ (explicit or implicit differential scheme), $\Phi_2 = 1/h_2$, $\Phi_3 = 1/h_3$ and

$$
R_{i,j+1}^{n} = \frac{0.5h_1 + 0.5h_3}{a_{i,j+1}} + \frac{0.5h_3}{a_{i,j+1}}, \ldots, R_{i-1,j}^{n} = \frac{0.5h_1}{a_{i-1,j}} + \frac{0.5h_3}{a_{i-1,j}} \tag{10}
$$

Definition of resistances (10) must be changed for $j = n$ (the last column of nodes), in particular

$$
R_{i,j+1}^{n} = \frac{0.5h_1 + 1}{a_{i,j}} \tag{11}
$$

while the others resistances are as previously.

Additionally in a place of $T_{*i,j}$, the cooling water temperature should be introduced. The approximation of remaining boundary conditions seems to be self-evident. The finite difference equations resulting from (8) and (9) are the linear ones both in case of explicit and implicit schemes.

3. Example of computations

The steel cast slab ($1 \times 0.2$ m) is considered. Thermophysical parameters of material are taken from [1]. The pouring temperature equals $T_p = 1550 ^\circ C$, pulling rate $w = 0.02$ m/s, heat transfer coefficients $\alpha = 1500$ ($x_3 \leq 0.7$), $\alpha = 1200$ ($0.7 < x_3 \leq 3.2$), $\alpha = 950$ ($3.2 < x_3 \leq 5.1$), $\alpha = 550$ ($5.1 < x_3 \leq 8.2$), $\alpha = 430$ W/m²K ($x_3 > 8.2$). The solution (pseudo-steady state) for $n = 5$, $m = 200$, $\Delta t = 3s$ obtained using the explicit scheme is shown in Figure 3.

![Figure 3: Cooling curves for successive $j$](image)

The computer program simulating the casting solidification has been also used for computations basing on the wandering cross section method (WCSM) proposed by Majchrzak and Mochnacki (e.g. [2]). The idea of WCSM consists in the analysis of thermal processes in the domain of casting lateral section which is ‘tied’ to the moving co-ordinate system. It turn out that the algorithm discussed here (in particular the presented method of substantial derivative approximation) and WCSM gives practically the same results. The further works concerning the subject of this paper should concentrate on the problems of proposed algorithm stability (the obtained solution is stable, but the problem of stability condition is open, as yet).

References

